

VECTOR MECHANICS for ENGINEERS

# STATICS

ELEVENTH EDITION

Beer • Johnston • Mazurek

A low-angle photograph of a tall, modern skyscraper with a glass facade, reflecting the sky and clouds. The building is the central focus, with other smaller skyscrapers visible in the background. The sky is blue with scattered white clouds.

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# Vector Mechanics For Engineers

Statics

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Eleventh Edition

# Vector Mechanics For Engineers

## Statics

**Ferdinand P. Beer**

Late of Lehigh University

**E. Russell Johnston, Jr.**

Late of University of Connecticut

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VECTOR MECHANICS FOR ENGINEERS: STATICS, ELEVENTH EDITION

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# Preface

## NEW!

The 11<sup>th</sup> edition has undergone a complete rewrite to modernize and streamline the language throughout the text.

## Objectives

A primary objective in a first course in mechanics is to help develop a student's ability first to analyze problems in a simple and logical manner, and then to apply basic principles to its solution. A strong conceptual understanding of these basic mechanics principles is essential for successfully solving mechanics problems. We hope that this text, as well as the proceeding volume, *Vector Mechanics for Engineers: Dynamics*, will help instructors achieve these goals.<sup>†</sup>

## General Approach

Vector analysis is introduced early in the text and is used in the presentation and discussion of the fundamental principles of mechanics. Vector methods are also used to solve many problems, particularly three-dimensional problems where these techniques result in a simpler and more concise solution. The emphasis in this text, however, remains on the correct understanding of the principles of mechanics and on their application to the solution of engineering problems, and vector analysis is presented chiefly as a convenient tool.<sup>‡</sup>

**Practical Applications Are Introduced Early.** One of the characteristics of the approach used in this book is that mechanics of *particles* is clearly separated from the mechanics of *rigid bodies*. This approach makes it possible to consider simple practical applications at an early stage and to postpone the introduction of the more difficult concepts. For example:

- In *Statics*, statics of particles is treated first (Chap. 2); after the rules of addition and subtraction of vectors are introduced, the principle of equilibrium of a particle is immediately applied to practical situations involving only concurrent forces. The statics of rigid bodies is considered in Chaps. 3 and 4. In Chap. 3, the vector and scalar products of two vectors are introduced and used to define the moment of a force about a point and about an axis. The presentation of these new concepts is followed by a thorough and rigorous discussion of equivalent systems of forces leading, in Chap. 4, to many practical applications involving the equilibrium of rigid bodies under general force systems.
- In *Dynamics*, the same division is observed. The basic concepts of force, mass, and acceleration, of work and energy, and of impulse and momentum are introduced and first applied to problems involving only particles. Thus, students can familiarize themselves with

### 2.2 ADDING FORCES BY COMPONENTS

In Sec. 2.1E, we described how to resolve a force into components. Here we discuss how to add forces by using their components, especially rectangular components. This method is often the most convenient way to add forces and, in practice, is the most common approach. (Note that we can readily extend the properties of vectors established in this section to the rectangular components of any vector quantity, such as velocity or momentum.)

#### 2.2A Rectangular Components of a Force: Unit Vectors

In many problems, it is useful to resolve a force into two components that are perpendicular to each other. Figure 2.14 shows a force  $F$  resolved into a component  $F_x$  along the  $x$  axis and a component  $F_y$  along the  $y$  axis. The parallelogram drawn to obtain the two components is a rectangle, and  $F_x$  and  $F_y$  are called **rectangular components**.

The  $x$  and  $y$  axes are usually chosen to be horizontal and vertical, respectively, as in Fig. 2.14; they may, however, be chosen in any two perpendicular directions, as shown in Fig. 2.15. In determining the

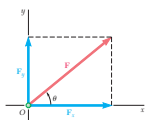


Fig. 2.14 Rectangular components of a force  $F$ .

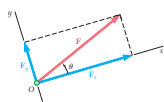


Fig. 2.15 Rectangular components of a force  $F$  for axes rotated away from horizontal and vertical.

<sup>†</sup>Both texts also are available in a single volume, *Vector Mechanics for Engineers: Statics and Dynamics*, eleventh edition.

<sup>‡</sup>In a parallel text, *Mechanics for Engineers: Statics*, fifth edition, the use of vector algebra is limited to the addition and subtraction of vectors.

the three basic methods used in dynamics and learn their respective advantages before facing the difficulties associated with the motion of rigid bodies.

**New Concepts Are Introduced in Simple Terms.** Since this text is designed for the first course in statics, new concepts are presented in simple terms and every step is explained in detail. On the other hand, by discussing the broader aspects of the problems considered, and by stressing methods of general applicability, a definite maturity of approach is achieved. For example, the concepts of partial constraints and statical indeterminacy are introduced early and are used throughout.

**Fundamental Principles Are Placed in the Context of Simple Applications.** The fact that mechanics is essentially a *deductive* science based on a few fundamental principles is stressed. Derivations have been presented in their logical sequence and with all the rigor warranted at this level. However, the learning process being largely *inductive*, simple applications are considered first. For example:

- The statics of particles precedes the statics of rigid bodies, and problems involving internal forces are postponed until Chap. 6.
- In Chap. 4, equilibrium problems involving only coplanar forces are considered first and solved by ordinary algebra, while problems involving three-dimensional forces and requiring the full use of vector algebra are discussed in the second part of the chapter.

**Systematic Problem-Solving Approach.** New to this edition of the text, all the sample problems are solved using the steps of *Strategy, Modeling, Analysis, and Reflect & Think*, or the “SMART” approach. This methodology is intended to give students confidence when approaching new problems, and students are encouraged to apply this approach in the solution of all assigned problems.

**Free-Body Diagrams Are Used Both to Solve Equilibrium Problems and to Express the Equivalence of Force Systems.** Free-body diagrams are introduced early, and their importance is emphasized throughout the text. They are used not only to solve equilibrium problems but also to express the equivalence of two systems of forces or, more generally, of two systems of vectors. The advantage of this approach becomes apparent in the study of the dynamics of rigid bodies, where it is used to solve three-dimensional as well as two-dimensional problems. By placing the emphasis on “free-body-diagram equations” rather than on the standard algebraic equations of motion, a more intuitive and more complete understanding of the fundamental principles of dynamics can be achieved. This approach, which was first introduced in 1962 in the first edition of *Vector Mechanics for Engineers*, has now gained wide acceptance among mechanics teachers in this country. It is, therefore, used in preference to the method of dynamic equilibrium and to the equations of motion in the solution of all sample problems in this book.

## 4.1 EQUILIBRIUM IN TWO DIMENSIONS

In the first part of this chapter, we consider the equilibrium of two-dimensional structures; i.e., we assume that the structure being analyzed and the forces applied to it are contained in the same plane. Clearly, the reactions needed to maintain the structure in the same position are also contained in this plane.

### 4.1A Reactions for a Two-Dimensional Structure

The reactions exerted on a two-dimensional structure fall into three categories that correspond to three types of **supports or connections**.

1. **Reactions Equivalent to a Force with a Known Line of Action.** Supports and connections causing reactions of this type include *rollers, rockers, frictionless surfaces, short links and cables, collars on frictionless rods, and frictionless pins in slots*. Each of these supports and connections can prevent motion in one direction only. Figure 4.1 shows these supports and connections together with the reactions they produce. Each reaction involves *one unknown*—specifically, the magnitude of the reaction. In problem solving, you should denote this magnitude by an appropriate letter. The line of action of the reaction is known and should be indicated clearly in the free-body diagram.

The sense of the reaction must be as shown in Fig. 4.1 for cases of a frictionless surface (toward the free body) or a cable (away from the free body). The reaction can be directed either way in the cases of double-track rollers, links, collars on rods, or pins in slots. Generally, we

NEW!

### A Four-Color Presentation Uses Color to Distinguish Vectors.

Color has been used, not only to enhance the quality of the illustrations, but also to help students distinguish among the various types of vectors they will encounter. While there was no intention to “color code” this text, the same color is used in any given chapter to represent vectors of the same type. Throughout *Statics*, for example, red is used exclusively to represent forces and couples, while position vectors are shown in blue and dimensions in black. This makes it easier for the students to identify the forces acting on a given particle or rigid body and to follow the discussion of sample problems and other examples given in the text.

### A Careful Balance Between SI and U.S. Customary Units Is Consistently Maintained.

Because of the current trend in the American government and industry to adopt the international system of units (SI metric units), the SI units most frequently used in mechanics are introduced in Chap. 1 and are used throughout the text. Approximately half of the sample problems and 60 percent of the homework problems are stated in these units, while the remainder are in U.S. customary units. The authors believe that this approach will best serve the need of students, who, as engineers, will have to be conversant with both systems of units.

It also should be recognized that using both SI and U.S. customary units entails more than the use of conversion factors. Since the SI system of units is an absolute system based on the units of time, length, and mass, whereas the U.S. customary system is a gravitational system based on the units of time, length, and force, different approaches are required for the solution of many problems. For example, when SI units are used, a body is generally specified by its mass expressed in kilograms; in most problems of statics it will be necessary to determine the weight of the body in newtons, and an additional calculation will be required for this purpose. On the other hand, when U.S. customary units are used, a body is specified by its weight in pounds and, in dynamics problems, an additional calculation will be required to determine its mass in slugs (or  $\text{lb}\cdot\text{s}^2/\text{ft}$ ). The authors, therefore, believe that problem assignments should include both systems of units.

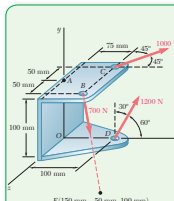
The *Instructor's and Solutions Manual* provides six different lists of assignments so that an equal number of problems stated in SI units and in U.S. customary units can be selected. If so desired, two complete lists of assignments can also be selected with up to 75 percent of the problems stated in SI units.

### Optional Sections Offer Advanced or Specialty Topics.

A large number of optional sections have been included. These sections are indicated by asterisks and thus are easily distinguished from those which form the core of the basic statics course. They can be omitted without prejudice to the understanding of the rest of the text.

Among the topics covered in these additional sections are the reduction of a system of forces to a wrench, applications to hydrostatics, equilibrium of cables, products of inertia and Mohr's circle, the determination of the principal axes and the mass moments of inertia of a body of arbitrary shape, and the method of virtual work. The sections on the inertia

**Sample Problem 3.10**



Three cables are attached to a bracket as shown. Replace the forces exerted by the cables with an equivalent force-couple system at A.

**STRATEGY:** First determine the relative position vectors drawn from point A to the points of application of the various forces and resolve the forces into rectangular components. Then sum the forces and moments.

**MODELING and ANALYSIS:** Note that  $F_B = (700 \text{ N})\mathbf{A}_{BE}$  where

$$\mathbf{A}_{BE} = \frac{\mathbf{BE}}{BE} = \frac{75\mathbf{i} - 150\mathbf{j} + 50\mathbf{k}}{175}$$

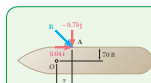
Using meters and newtons, the position and force vectors are

$$\begin{aligned} \mathbf{r}_{AB} &= 0.075\mathbf{i} + 0.050\mathbf{k} & \mathbf{F}_A &= 300\mathbf{i} - 600\mathbf{j} + 200\mathbf{k} \\ \mathbf{r}_{AC} &= 0.075\mathbf{i} - 0.050\mathbf{k} & \mathbf{F}_C &= 707\mathbf{i} & & - 707\mathbf{k} \\ \mathbf{r}_{AD} &= 0.100\mathbf{i} - 0.100\mathbf{j} & \mathbf{F}_D &= 600\mathbf{i} + 1039\mathbf{j} \end{aligned}$$

The force-couple system at A equivalent to the given forces consists of a force  $\mathbf{R} = \sum \mathbf{F}$  and a couple  $\mathbf{M}_A^R = \sum (\mathbf{r} \times \mathbf{F})$ . Obtain the force  $\mathbf{R}$  by adding respectively the x, y, and z components of the forces:

$$\mathbf{R} = \sum \mathbf{F} = (1607 \text{ N})\mathbf{i} + (439 \text{ N})\mathbf{j} - (507 \text{ N})\mathbf{k} \quad \leftarrow$$

(continued)



**Fig. 3** Point of application of single tugboat to create same effect as given force system.

**Remark:** Since all the forces are contained in the plane of the figure, you would expect the sum of their moments to be perpendicular to that plane. Note that you could obtain the moment of each force component directly from the diagram by first forming the product of its magnitude and perpendicular distance to O and then assigning to this product a positive or a negative sign, depending upon the sense of the moment.

**b. Single Tugboat.** The force exerted by a single tugboat must be equal to  $\mathbf{R}$ , and its point of application A must be such that the moment of  $\mathbf{R}$  about O is equal to  $\mathbf{M}_O^R$  (Fig. 3). Observing that the position vector of A is

$$\mathbf{r} = x\mathbf{i} + 70\mathbf{j}$$

you have

$$\begin{aligned} \mathbf{r} \times \mathbf{R} &= \mathbf{M}_O^R \\ (x\mathbf{i} + 70\mathbf{j}) \times (0.04\mathbf{i} - 9.79\mathbf{j}) &= -103\mathbf{k} \\ -x(0.79)\mathbf{k} &= 63\mathbf{k} = -103\mathbf{k} & x &= -41.1 \text{ ft} \quad \leftarrow \end{aligned}$$

**REFLECT and THINK:** Reducing the given situation to that of a single force makes it easier to visualize the overall effect of the tugboats in maneuvering the ocean liner. But in practical terms, having four boats applying force allows for greater control in slowing and turning a large ship in a crowded harbor.

properties of three-dimensional bodies are primarily intended for students who will later study in dynamics the three-dimensional motion of rigid bodies.

The material presented in the text and most of the problems require no previous mathematical knowledge beyond algebra, trigonometry, and elementary calculus; all the elements of vector algebra necessary to the understanding of the text are carefully presented in Chaps. 2 and 3. In general, a greater emphasis is placed on the correct understanding of the basic mathematical concepts involved than on the nimble manipulation of mathematical formulas. In this connection, it should be mentioned that the determination of the centroids of composite areas precedes the calculation of centroids by integration, thus making it possible to establish the concept of the moment of an area firmly before introducing the use of integration.

# Guided Tour

**Chapter Introduction.** Each chapter begins with a list of learning objectives and an outline that previews chapter topics. An introductory section describes the material to be covered in simple terms, and how it will be applied to the solution of engineering problems.

**Chapter Lessons.** The body of the text is divided into sections, each consisting of one or more sub-sections, several sample problems, and a large number of end-of-section problems for students to solve. Each section corresponds to a well-defined topic and generally can be covered in one lesson. In a number of cases, however, the instructor will find it desirable to devote more than one lesson to a given topic. *The Instructor's and Solutions Manual* contains suggestions on the coverage of each lesson.

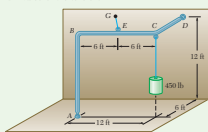
NEW!

**Concept Applications.** Concept Applications are used within selected theory sections to amplify certain topics, and they are designed to reinforce the specific material being presented and facilitate its understanding.

**Sample Problems.** The Sample Problems are set up in much the same form that students will use when solving assigned problems, and they employ the SMART problem-solving methodology that students are encouraged to use in the solution of their assigned problems. They thus serve the double purpose of amplifying the text and demonstrating the type of neat and orderly work that students should cultivate in their own solutions. In addition, in-problem references and captions have been added to the sample problem figures for contextual linkage to the step-by-step solution.

## Sample Problem 4.10

A 450-lb load hangs from the corner  $C$  of a rigid piece of pipe  $ABCD$  that has been bent as shown. The pipe is supported by ball-and-socket joints  $A$  and  $D$ , which are fastened, respectively, to the floor and to a vertical wall, and by a cable attached at the midpoint  $E$  of the portion  $BC$  of the pipe and at a point  $G$  on the wall. Determine (a) where  $G$  should be located if the tension in the cable is to be minimum, (b) the corresponding minimum value of the tension.



**STRATEGY:** Draw the free-body diagram of the pipe showing the reactions at  $A$  and  $D$ . Isolate the unknown tension  $T$  and the known weight  $W$  by summing moments about the diagonal line  $AD$ , and compute values from the equilibrium equations.

### MODELING and ANALYSIS:

**Free-Body Diagram.** The free-body diagram of the pipe includes the load  $W = (-450 \text{ lb})\mathbf{j}$ , the reactions at  $A$  and  $D$ , and the force  $T$  exerted by the cable (Fig. 1). To eliminate the reactions at  $A$  and  $D$  from the computations, take the sum of the moments of the forces about the line  $AD$  and set it equal to zero. Denote the unit vector along  $AD$  by  $\mathbf{A}$ , which enables you to write

$$\sum M_{AD} = 0: \quad \mathbf{A} \cdot (\overline{AE} \times T) + \mathbf{A} \cdot (\overline{AC} \times W) = 0 \quad (1)$$

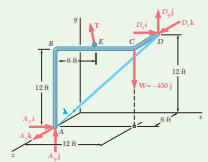


Fig. 1 Free-body diagram of pipe.

**Solving Problems on Your Own.** A section entitled *Solving Problems on Your Own* is included for each lesson, between the sample problems and the problems to be assigned. The purpose of these sections is to help students organize in their own minds the preceding theory of the text and the solution methods of the sample problems so that they can more successfully solve the homework problems. Also included in these sections are specific suggestions and strategies that will enable the students to more efficiently attack any assigned problems.

**Homework Problem Sets.** Most of the problems are of a practical nature and should appeal to engineering students. They are primarily designed, however, to illustrate the material presented in the text and to help students understand the principles of mechanics. The problems are grouped according to the portions of material they illustrate and, in general, are arranged in order of increasing difficulty. Problems requiring special attention are indicated by asterisks. Answers to 70 percent of the problems are given at the end of the book. Problems for which the answers are given are set in straight type in the text, while problems for which no answer is given are set in italic and red font color.

NEW!

Over 300 of the homework problems in the text are new or revised.


**Chapter Review and Summary.** Each chapter ends with a review and summary of the material covered in that chapter. Marginal notes are used to help students organize their review work, and cross-references have been included to help them find the portions of material requiring their special attention.

**Review Problems.** A set of review problems is included at the end of each chapter. These problems provide students further opportunity to apply the most important concepts introduced in the chapter.

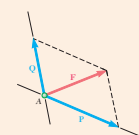
## Review and Summary

In this chapter, we have studied the effect of forces on particles, i.e., on bodies of such shape and size that we may assume all forces acting on them apply at the same point.

**Resultant of Two Forces**  
 Forces are *vector quantities*; they are characterized by a point of application, a magnitude, and a direction, and they add according to the parallelogram law (Fig. 2.30). We can determine the magnitude and direction of the resultant **R** of two forces **P** and **Q** either graphically or by trigonometry using the law of cosines and the law of sines (Sample Prob. 2.1).



**Components of a Force**  
 Any given force acting on a particle can be resolved into two or more components, i.e., it can be replaced by two or more forces that have the same effect on the particle. A force **F** can be resolved into two components **P** and **Q** by drawing a parallelogram with **F** for its diagonal; the components **P** and **Q** are then represented by the two adjacent sides of the parallelogram (Fig. 2.31). Again, we can determine the components either graphically or by trigonometry [Sec. 2.1E].



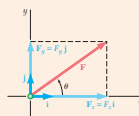
**Rectangular Components; Unit Vectors**  
 A force **F** is resolved into two rectangular components if its components **F<sub>x</sub>** and **F<sub>y</sub>** are perpendicular to each other and are directed along the coordinate axes (Fig. 2.32). Introducing the unit vectors **i** and **j** along the *x* and *y* axes, respectively, we can write the components and the vector as [Sec. 2.2A]

$$\mathbf{F}_x = F_x \mathbf{i} \quad \mathbf{F}_y = F_y \mathbf{j} \quad (2.6)$$

and

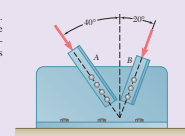
$$\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j} \quad (2.7)$$

where  $F_x$  and  $F_y$  are the *scalar components* of **F**. These components, which can be positive or negative, are defined by the relations

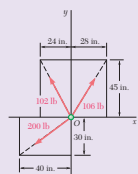
$$F_x = F \cos \theta \quad F_y = F \sin \theta \quad (2.8)$$


## Review Problems

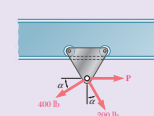
**2.127** Two structural members **A** and **B** are bolted to a bracket as shown. Knowing that both members are in compression and that the force is 15 kN in member **A** and 10 kN in member **B**, determine by trigonometry the magnitude and direction of the resultant of the forces applied to the bracket by members **A** and **B**.



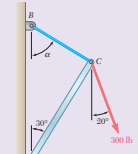
**2.128** Determine the *x* and *y* components of each of the forces shown.



**2.129** A hoist trolley is subjected to the three forces shown. Knowing that  $\alpha = 40^\circ$ , determine (a) the required magnitude of the force **P** if the resultant of the three forces is to be vertical, (b) the corresponding magnitude of the resultant.



**2.130** Knowing that  $\alpha = 55^\circ$  and that boom **AC** exerts on pin **C** a force directed along line **AC**, determine (a) the magnitude of that force, (b) the tension in cable **BC**.



**Computer Problems.** Accessible through Connect are problem sets for each chapter that are designed to be solved with computational software. Many of these problems are relevant to the design process; they may involve the analysis of a structure for various configurations and loadings of the structure, or the determination of the equilibrium positions of a given mechanism that may require an iterative method of solution. Developing the algorithm required to solve a given mechanics problem will benefit the students in two different ways: (1) it will help them gain a better understanding of the mechanics principles involved; (2) it will provide them with an opportunity to apply their computer skills to the solution of a meaningful engineering problem.



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# List of Symbols

$a$	Constant; radius; distance	$U$	Work
<b>A, B, C, . . .</b>	Reactions at supports and connections	<b>V</b>	Vector product; shearing force
$A, B, C, . . .$	Points	$V$	Volume; potential energy; shear
$A$	Area	$w$	Load per unit length
$b$	Width; distance	<b>W, W</b>	Weight; load
$c$	Constant	$x, y, z$	Rectangular coordinates; distances
$C$	Centroid	$\bar{x}, \bar{y}, \bar{z}$	Rectangular coordinates of centroid or center of gravity
$d$	Distance	$\alpha, \beta, \gamma$	Angles
$e$	Base of natural logarithms	$\gamma$	Specific weight
<b>F</b>	Force; friction force	$\delta$	Elongation
$g$	Acceleration of gravity	$\delta\mathbf{r}$	Virtual displacement
$G$	Center of gravity; constant of gravitation	$\delta U$	Virtual work
$h$	Height; sag of cable	$\lambda$	Unit vector along a line
<b>i, j, k</b>	Unit vectors along coordinate axes	$\eta$	Efficiency
$I, I_x, . . .$	Moments of inertia	$\theta$	Angular coordinate; angle; polar coordinate
$\bar{I}$	Centroidal moment of inertia	$\mu$	Coefficient of friction
$I_{xy}, . . .$	Products of inertia	$\rho$	Density
$J$	Polar moment of inertia	$\phi$	Angle of friction; angle
$k$	Spring constant		
$k_x, k_y, k_O$	Radii of gyration		
$\bar{k}$	Centroidal radius of gyration		
$l$	Length		
$L$	Length; span		
$m$	Mass		
<b>M</b>	Couple; moment		
$\mathbf{M}_O$	Moment about point $O$		
$\mathbf{M}_O^R$	Moment resultant about point $O$		
$M$	Magnitude of couple or moment; mass of earth		
$M_{OL}$	Moment about axis $OL$		
<b>N</b>	Normal component of reaction		
$O$	Origin of coordinates		
$p$	Pressure		
<b>P</b>	Force; vector		
<b>Q</b>	Force; vector		
<b>r</b>	Position vector		
$r$	Radius; distance; polar coordinate		
<b>R</b>	Resultant force; resultant vector; reaction		
$R$	Radius of earth		
<b>s</b>	Position vector		
$s$	Length of arc; length of cable		
<b>S</b>	Force; vector		
$t$	Thickness		
<b>T</b>	Force		
$T$	Tension		

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# 1

## Introduction

The tallest skyscraper in the Western Hemisphere, One World Trade Center is a prominent feature of the New York City skyline. From its foundation to its structural components and mechanical systems, the design and operation of the tower is based on the fundamentals of engineering mechanics.

## Introduction

- 1.1 WHAT IS MECHANICS?
- 1.2 FUNDAMENTAL CONCEPTS AND PRINCIPLES
- 1.3 SYSTEMS OF UNITS
- 1.4 CONVERTING BETWEEN TWO SYSTEMS OF UNITS
- 1.5 METHOD OF SOLVING PROBLEMS
- 1.6 NUMERICAL ACCURACY

## Objectives

- **Define** the science of mechanics and examine its fundamental principles.
- **Discuss** and compare the International System of Units and U.S. Customary Units.
- **Discuss** how to approach the solution of mechanics problems, and introduce the SMART problem-solving methodology.
- **Examine** factors that govern numerical accuracy in the solution of a mechanics problem.

### 1.1 What is Mechanics?

Mechanics is defined as the science that describes and predicts the conditions of rest or motion of bodies under the action of forces. It consists of the mechanics of *rigid bodies*, mechanics of *deformable bodies*, and mechanics of *fluids*.

The mechanics of rigid bodies is subdivided into **statics** and **dynamics**. Statics deals with bodies at rest; dynamics deals with bodies in motion. In this text, we assume bodies are perfectly rigid. In fact, actual structures and machines are never absolutely rigid; they deform under the loads to which they are subjected. However, because these deformations are usually small, they do not appreciably affect the conditions of equilibrium or the motion of the structure under consideration. They are important, though, as far as the resistance of the structure to failure is concerned. Deformations are studied in a course in mechanics of materials, which is part of the mechanics of deformable bodies. The third division of mechanics, the mechanics of fluids, is subdivided into the study of *incompressible fluids* and of *compressible fluids*. An important subdivision of the study of incompressible fluids is *hydraulics*, which deals with applications involving water.

Mechanics is a physical science, since it deals with the study of physical phenomena. However, some teachers associate mechanics with mathematics, whereas many others consider it as an engineering subject. Both these views are justified in part. Mechanics is the foundation of most engineering sciences and is an indispensable prerequisite to their study. However, it does not have the *empiricism* found in some engineering sciences, i.e., it does not rely on experience or observation alone. The rigor of mechanics and the emphasis it places on deductive reasoning makes it resemble mathematics. However, mechanics is not an *abstract* or even a *pure* science; it is an *applied* science.

The purpose of mechanics is to explain and predict physical phenomena and thus to lay the foundations for engineering applications. You need to know statics to determine how much force will be exerted on a point in a bridge design and whether the structure can withstand that force. Determining the force a dam needs to withstand from the water in a river requires statics. You need statics to calculate how much weight a crane can lift, how much force a locomotive needs to pull a freight train, or how



much force a circuit board in a computer can withstand. The concepts of dynamics enable you to analyze the flight characteristics of a jet, design a building to resist earthquakes, and mitigate shock and vibration to passengers inside a vehicle. The concepts of dynamics enable you to calculate how much force you need to send a satellite into orbit, accelerate a 200,000-ton cruise ship, or design a toy truck that doesn't break. You will not learn how to do these things in this course, but the ideas and methods you learn here will be the underlying basis for the engineering applications you will learn in your work.

## 1.2 Fundamental Concepts and Principles

Although the study of mechanics goes back to the time of Aristotle (384–322 B.C.) and Archimedes (287–212 B.C.), not until Newton (1642–1727) did anyone develop a satisfactory formulation of its fundamental principles. These principles were later modified by d'Alembert, Lagrange, and Hamilton. Their validity remained unchallenged until Einstein formulated his **theory of relativity** (1905). Although its limitations have now been recognized, **newtonian mechanics** still remains the basis of today's engineering sciences.

The basic concepts used in mechanics are *space*, *time*, *mass*, and *force*. These concepts cannot be truly defined; they should be accepted on the basis of our intuition and experience and used as a mental frame of reference for our study of mechanics.

The concept of **space** is associated with the position of a point  $P$ . We can define the position of  $P$  by providing three lengths measured from a certain reference point, or *origin*, in three given directions. These lengths are known as the *coordinates* of  $P$ .

To define an event, it is not sufficient to indicate its position in space. We also need to specify the **time** of the event.

We use the concept of **mass** to characterize and compare bodies on the basis of certain fundamental mechanical experiments. Two bodies of the same mass, for example, are attracted by the earth in the same manner; they also offer the same resistance to a change in translational motion.

A **force** represents the action of one body on another. A force can be exerted by actual contact, like a push or a pull, or at a distance, as in the case of gravitational or magnetic forces. A force is characterized by its *point of application*, its *magnitude*, and its *direction*; a force is represented by a *vector* (Sec. 2.1B).

In newtonian mechanics, space, time, and mass are absolute concepts that are independent of each other. (This is not true in **relativistic mechanics**, where the duration of an event depends upon its position and the mass of a body varies with its velocity.) On the other hand, the concept of force is not independent of the other three. Indeed, one of the fundamental principles of newtonian mechanics listed below is that the resultant force acting on a body is related to the mass of the body and to the manner in which its velocity varies with time.

In this text, you will study the conditions of rest or motion of particles and rigid bodies in terms of the four basic concepts we have introduced. By **particle**, we mean a very small amount of matter, which we



assume occupies a single point in space. A **rigid body** consists of a large number of particles occupying fixed positions with respect to one another. The study of the mechanics of particles is clearly a prerequisite to that of rigid bodies. Besides, we can use the results obtained for a particle directly in a large number of problems dealing with the conditions of rest or motion of actual bodies.

The study of elementary mechanics rests on six fundamental principles, based on experimental evidence.

- **The Parallelogram Law for the Addition of Forces.** Two forces acting on a particle may be replaced by a single force, called their *resultant*, obtained by drawing the diagonal of the parallelogram with sides equal to the given forces (Sec. 2.1A).
- **The Principle of Transmissibility.** The conditions of equilibrium or of motion of a rigid body remain unchanged if a force acting at a given point of the rigid body is replaced by a force of the same magnitude and same direction, but acting at a different point, provided that the two forces have the same line of action (Sec. 3.1B).
- **Newton's Three Laws of Motion.** Formulated by Sir Isaac Newton in the late seventeenth century, these laws can be stated as follows:

**FIRST LAW.** If the resultant force acting on a particle is zero, the particle remains at rest (if originally at rest) or moves with constant speed in a straight line (if originally in motion) (Sec. 2.3B).

**SECOND LAW.** If the resultant force acting on a particle is not zero, the particle has an acceleration proportional to the magnitude of the resultant and in the direction of this resultant force.

As you will see in Sec. 12.1, this law can be stated as

$$\mathbf{F} = m\mathbf{a} \quad (1.1)$$

where  $\mathbf{F}$ ,  $m$ , and  $\mathbf{a}$  represent, respectively, the resultant force acting on the particle, the mass of the particle, and the acceleration of the particle expressed in a consistent system of units.

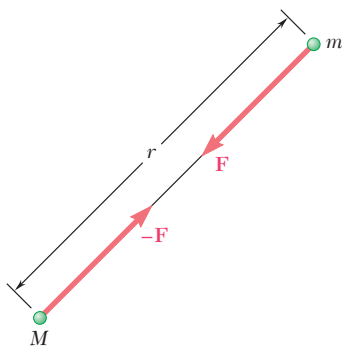
**THIRD LAW.** The forces of action and reaction between bodies in contact have the same magnitude, same line of action, and opposite sense (Ch. 6, Introduction).

- **Newton's Law of Gravitation.** Two particles of mass  $M$  and  $m$  are mutually attracted with equal and opposite forces  $\mathbf{F}$  and  $-\mathbf{F}$  of magnitude  $F$  (Fig. 1.1), given by the formula

$$F = G \frac{Mm}{r^2} \quad (1.2)$$

where  $r$  = the distance between the two particles and  $G$  = a universal constant called the *constant of gravitation*. Newton's law of gravitation introduces the idea of an action exerted at a distance and extends the range of application of Newton's third law: the action  $\mathbf{F}$  and the reaction  $-\mathbf{F}$  in Fig. 1.1 are equal and opposite, and they have the same line of action.

A particular case of great importance is that of the attraction of the earth on a particle located on its surface. The force  $\mathbf{F}$  exerted by the earth on the particle is defined as the **weight**  $\mathbf{W}$  of the particle. Suppose we set



**Fig. 1.1** From Newton's law of gravitation, two particles of masses  $M$  and  $m$  exert forces upon each other of equal magnitude, opposite direction, and the same line of action. This also illustrates Newton's third law of motion.

$M$  equal to the mass of the earth,  $m$  equal to the mass of the particle, and  $r$  equal to the earth's radius  $R$ . Then introducing the constant

$$g = \frac{GM}{R^2} \quad (1.3)$$

we can express the magnitude  $W$  of the weight of a particle of mass  $m$  as<sup>†</sup>

$$W = mg \quad (1.4)$$

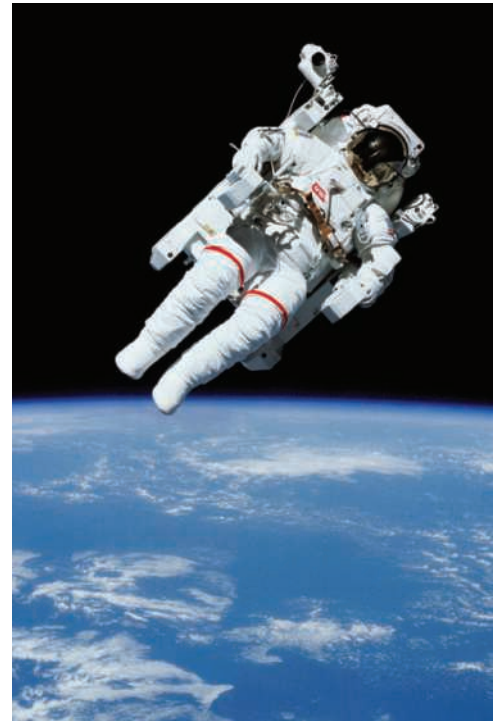
The value of  $R$  in formula (1.3) depends upon the elevation of the point considered; it also depends upon its latitude, since the earth is not truly spherical. The value of  $g$  therefore varies with the position of the point considered. However, as long as the point actually remains on the earth's surface, it is sufficiently accurate in most engineering computations to assume that  $g$  equals  $9.81 \text{ m/s}^2$  or  $32.2 \text{ ft/s}^2$ .

The principles we have just listed will be introduced in the course of our study of mechanics as they are needed. The statics of particles carried out in Chap. 2 will be based on the parallelogram law of addition and on Newton's first law alone. We introduce the principle of transmissibility in Chap. 3 as we begin the study of the statics of rigid bodies, and we bring in Newton's third law in Chap. 6 as we analyze the forces exerted on each other by the various members forming a structure. We introduce Newton's second law and Newton's law of gravitation in dynamics. We will then show that Newton's first law is a particular case of Newton's second law (Sec. 12.1) and that the principle of transmissibility could be derived from the other principles and thus eliminated (Sec. 16.1D). In the meantime, however, Newton's first and third laws, the parallelogram law of addition, and the principle of transmissibility will provide us with the necessary and sufficient foundation for the entire study of the statics of particles, rigid bodies, and systems of rigid bodies.

As noted earlier, the six fundamental principles listed previously are based on experimental evidence. Except for Newton's first law and the principle of transmissibility, they are independent principles that cannot be derived mathematically from each other or from any other elementary physical principle. On these principles rests most of the intricate structure of newtonian mechanics. For more than two centuries, engineers have solved a tremendous number of problems dealing with the conditions of rest and motion of rigid bodies, deformable bodies, and fluids by applying these fundamental principles. Many of the solutions obtained could be checked experimentally, thus providing a further verification of the principles from which they were derived. Only in the twentieth century has Newton's mechanics found to be at fault, in the study of the motion of atoms and the motion of the planets, where it must be supplemented by the theory of relativity. On the human or engineering scale, however, where velocities are small compared with the speed of light, Newton's mechanics have yet to be disproved.

## 1.3 Systems of Units

Associated with the four fundamental concepts just discussed are the so-called *kinetic units*, i.e., the units of *length*, *time*, *mass*, and *force*. These units cannot be chosen independently if Eq. (1.1) is to be satisfied.



**Photo 1.1** When in orbit of the earth, people and objects are said to be *weightless* even though the gravitational force acting is approximately 90% of that experienced on the surface of the earth. This apparent contradiction will be resolved in Chapter 12 when we apply Newton's second law to the motion of particles.

<sup>†</sup>A more accurate definition of the weight  $\mathbf{W}$  should take into account the earth's rotation.

Three of the units may be defined arbitrarily; we refer to them as **basic units**. The fourth unit, however, must be chosen in accordance with Eq. (1.1) and is referred to as a **derived unit**. Kinetic units selected in this way are said to form a **consistent system of units**.

**International System of Units (SI Units).<sup>†</sup>** In this system, which will be in universal use after the United States has completed its conversion to SI units, the base units are the units of length, mass, and time, and they are called, respectively, the **meter** (m), the **kilogram** (kg), and the **second** (s). All three are arbitrarily defined. The second was originally chosen to represent 1/86 400 of the mean solar day, but it is now defined as the duration of 9 192 631 770 cycles of the radiation corresponding to the transition between two levels of the fundamental state of the cesium-133 atom. The meter, originally defined as one ten-millionth of the distance from the equator to either pole, is now defined as 1 650 763.73 wavelengths of the orange-red light corresponding to a certain transition in an atom of krypton-86. (The newer definitions are much more precise and with today's modern instrumentation, are easier to verify as a standard.) The kilogram, which is approximately equal to the mass of 0.001 m<sup>3</sup> of water, is defined as the mass of a platinum-iridium standard kept at the International Bureau of Weights and Measures at Sèvres, near Paris, France. The unit of force is a derived unit. It is called the **newton** (N) and is defined as the force that gives an acceleration of 1 m/s<sup>2</sup> to a body of mass 1 kg (Fig. 1.2). From Eq. (1.1), we have

$$1 \text{ N} = (1 \text{ kg})(1 \text{ m/s}^2) = 1 \text{ kg}\cdot\text{m/s}^2 \quad (1.5)$$

The SI units are said to form an *absolute* system of units. This means that the three base units chosen are independent of the location where measurements are made. The meter, the kilogram, and the second may be used anywhere on the earth; they may even be used on another planet and still have the same significance.

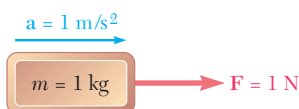
The *weight* of a body, or the *force of gravity* exerted on that body, like any other force, should be expressed in newtons. From Eq. (1.4), it follows that the weight of a body of mass 1 kg (Fig. 1.3) is

$$\begin{aligned} W &= mg \\ &= (1 \text{ kg})(9.81 \text{ m/s}^2) \\ &= 9.81 \text{ N} \end{aligned}$$

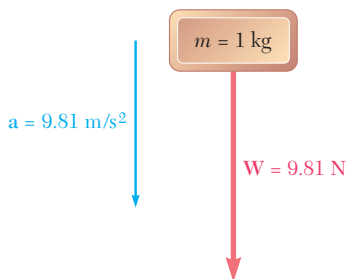
Multiples and submultiples of the fundamental SI units are denoted through the use of the prefixes defined in Table 1.1. The multiples and submultiples of the units of length, mass, and force most frequently used in engineering are, respectively, the *kilometer* (km) and the *millimeter* (mm); the *megagram*<sup>‡</sup> (Mg) and the *gram* (g); and the *kilonewton* (kN). According to Table 1.1, we have

$$\begin{aligned} 1 \text{ km} &= 1000 \text{ m} & 1 \text{ mm} &= 0.001 \text{ m} \\ 1 \text{ Mg} &= 1000 \text{ kg} & 1 \text{ g} &= 0.001 \text{ kg} \\ 1 \text{ kN} &= 1000 \text{ N} \end{aligned}$$

The conversion of these units into meters, kilograms, and newtons, respectively, can be effected by simply moving the decimal point three places



**Fig. 1.2** A force of 1 newton applied to a body of mass 1 kg provides an acceleration of 1 m/s<sup>2</sup>.



**Fig. 1.3** A body of mass 1 kg experiencing an acceleration due to gravity of 9.81 m/s<sup>2</sup> has a weight of 9.81 N.

<sup>†</sup>SI stands for *Système International d'Unités* (French)

<sup>‡</sup>Also known as a *metric ton*.

**Table 1.1 SI Prefixes**

Multiplication Factor	Prefix <sup>†</sup>	Symbol
1 000 000 000 000 = 10 <sup>12</sup>	tera	T
1 000 000 000 = 10 <sup>9</sup>	giga	G
1 000 000 = 10 <sup>6</sup>	mega	M
1 000 = 10 <sup>3</sup>	kilo	k
100 = 10 <sup>2</sup>	hecto <sup>‡</sup>	h
10 = 10 <sup>1</sup>	deka <sup>‡</sup>	da
0.1 = 10 <sup>-1</sup>	deci <sup>‡</sup>	d
0.01 = 10 <sup>-2</sup>	centi <sup>‡</sup>	c
0.001 = 10 <sup>-3</sup>	milli	m
0.000 001 = 10 <sup>-6</sup>	micro	μ
0.000 000 001 = 10 <sup>-9</sup>	nano	n
0.000 000 000 001 = 10 <sup>-12</sup>	pico	p
0.000 000 000 000 001 = 10 <sup>-15</sup>	femto	f
0.000 000 000 000 000 001 = 10 <sup>-18</sup>	atto	a

<sup>†</sup>The first syllable of every prefix is accented, so that the prefix retains its identity. Thus, the preferred pronunciation of kilometer places the accent on the first syllable, not the second.

<sup>‡</sup>The use of these prefixes should be avoided, except for the measurement of areas and volumes and for the nontechnical use of centimeter, as for body and clothing measurements.

to the right or to the left. For example, to convert 3.82 km into meters, move the decimal point three places to the right:

$$3.82 \text{ km} = 3820 \text{ m}$$

Similarly, to convert 47.2 mm into meters, move the decimal point three places to the left:

$$47.2 \text{ mm} = 0.0472 \text{ m}$$

Using engineering notation, you can also write

$$\begin{aligned} 3.82 \text{ km} &= 3.82 \times 10^3 \text{ m} \\ 47.2 \text{ mm} &= 47.2 \times 10^{-3} \text{ m} \end{aligned}$$

The multiples of the unit of time are the *minute* (min) and the *hour* (h). Since 1 min = 60 s and 1 h = 60 min = 3600 s, these multiples cannot be converted as readily as the others.

By using the appropriate multiple or submultiple of a given unit, you can avoid writing very large or very small numbers. For example, it is usually simpler to write 427.2 km rather than 427 200 m and 2.16 mm rather than 0.002 16 m.<sup>†</sup>

**Units of Area and Volume.** The unit of area is the *square meter* (m<sup>2</sup>), which represents the area of a square of side 1 m; the unit of volume is the *cubic meter* (m<sup>3</sup>), which is equal to the volume of a cube of side 1 m. In order to avoid exceedingly small or large numerical values when computing areas and volumes, we use systems of subunits obtained by respectively squaring and cubing not only the millimeter, but also two intermediate

<sup>†</sup>Note that when more than four digits appear on either side of the decimal point to express a quantity in SI units—as in 427 000 m or 0.002 16 m—use spaces, never commas, to separate the digits into groups of three. This practice avoids confusion with the comma used in place of a decimal point, which is the convention in many countries.

submultiples of the meter: the *decimeter* (dm) and the *centimeter* (cm). By definition,

$$\begin{aligned}1 \text{ dm} &= 0.1 \text{ m} = 10^{-1} \text{ m} \\1 \text{ cm} &= 0.01 \text{ m} = 10^{-2} \text{ m} \\1 \text{ mm} &= 0.001 \text{ m} = 10^{-3} \text{ m}\end{aligned}$$

Therefore, the submultiples of the unit of area are

$$\begin{aligned}1 \text{ dm}^2 &= (1 \text{ dm})^2 = (10^{-1} \text{ m})^2 = 10^{-2} \text{ m}^2 \\1 \text{ cm}^2 &= (1 \text{ cm})^2 = (10^{-2} \text{ m})^2 = 10^{-4} \text{ m}^2 \\1 \text{ mm}^2 &= (1 \text{ mm})^2 = (10^{-3} \text{ m})^2 = 10^{-6} \text{ m}^2\end{aligned}$$

Similarly, the submultiples of the unit of volume are

$$\begin{aligned}1 \text{ dm}^3 &= (1 \text{ dm})^3 = (10^{-1} \text{ m})^3 = 10^{-3} \text{ m}^3 \\1 \text{ cm}^3 &= (1 \text{ cm})^3 = (10^{-2} \text{ m})^3 = 10^{-6} \text{ m}^3 \\1 \text{ mm}^3 &= (1 \text{ mm})^3 = (10^{-3} \text{ m})^3 = 10^{-9} \text{ m}^3\end{aligned}$$

Note that when measuring the volume of a liquid, the cubic decimeter ( $\text{dm}^3$ ) is usually referred to as a *liter* (L).

Table 1.2 shows other derived SI units used to measure the moment of a force, the work of a force, etc. Although we will introduce these units in later chapters as they are needed, we should note an important rule at

**Table 1.2 Principal SI Units Used in Mechanics**

Quantity	Unit	Symbol	Formula
Acceleration	Meter per second squared	...	$\text{m/s}^2$
Angle	Radian	rad	†
Angular acceleration	Radian per second squared	...	$\text{rad/s}^2$
Angular velocity	Radian per second	...	$\text{rad/s}$
Area	Square meter	...	$\text{m}^2$
Density	Kilogram per cubic meter	...	$\text{kg/m}^3$
Energy	Joule	J	$\text{N}\cdot\text{m}$
Force	Newton	N	$\text{kg}\cdot\text{m/s}^2$
Frequency	Hertz	Hz	$\text{s}^{-1}$
Impulse	Newton-second	...	$\text{kg}\cdot\text{m/s}$
Length	Meter	m	‡
Mass	Kilogram	kg	‡
Moment of a force	Newton-meter	...	$\text{N}\cdot\text{m}$
Power	Watt	W	$\text{J/s}$
Pressure	Pascal	Pa	$\text{N/m}^2$
Stress	Pascal	Pa	$\text{N/m}^2$
Time	Second	s	‡
Velocity	Meter per second	...	$\text{m/s}$
Volume			
Solids	Cubic meter	...	$\text{m}^3$
Liquids	Liter	L	$10^{-3} \text{ m}^3$
Work	Joule	J	$\text{N}\cdot\text{m}$

†Supplementary unit (1 revolution =  $2\pi$  rad =  $360^\circ$ ).

‡Base unit.

this time: When a derived unit is obtained by dividing a base unit by another base unit, you may use a prefix in the numerator of the derived unit, but not in its denominator. For example, the constant  $k$  of a spring that stretches 20 mm under a load of 100 N is expressed as

$$k = \frac{100 \text{ N}}{20 \text{ mm}} = \frac{100 \text{ N}}{0.020 \text{ m}} = 5000 \text{ N/m} \text{ or } k = 5 \text{ kN/m}$$

but never as  $k = 5 \text{ N/mm}$ .

**U.S. Customary Units.** Most practicing American engineers still commonly use a system in which the base units are those of length, force, and time. These units are, respectively, the *foot* (ft), the *pound* (lb), and the *second* (s). The second is the same as the corresponding SI unit. The foot is defined as 0.3048 m. The pound is defined as the *weight* of a platinum standard, called the *standard pound*, which is kept at the National Institute of Standards and Technology outside Washington D.C., the mass of which is 0.453 592 43 kg. Since the weight of a body depends upon the earth's gravitational attraction, which varies with location, the standard pound should be placed at sea level and at a latitude of  $45^\circ$  to properly define a force of 1 lb. Clearly the U.S. customary units do not form an absolute system of units. Because they depend upon the gravitational attraction of the earth, they form a *gravitational* system of units.

Although the standard pound also serves as the unit of mass in commercial transactions in the United States, it cannot be used that way in engineering computations, because such a unit would not be consistent with the base units defined in the preceding paragraph. Indeed, when acted upon by a force of 1 lb—that is, when subjected to the force of gravity—the standard pound has the acceleration due to gravity,  $g = 32.2 \text{ ft/s}^2$  (Fig. 1.4), not the unit acceleration required by Eq. (1.1). The unit of mass consistent with the foot, the pound, and the second is the mass that receives an acceleration of  $1 \text{ ft/s}^2$  when a force of 1 lb is applied to it (Fig. 1.5). This unit, sometimes called a *slug*, can be derived from the equation  $F = ma$  after substituting 1 lb for  $F$  and  $1 \text{ ft/s}^2$  for  $a$ . We have

$$F = ma \quad 1 \text{ lb} = (1 \text{ slug})(1 \text{ ft/s}^2)$$

This gives us

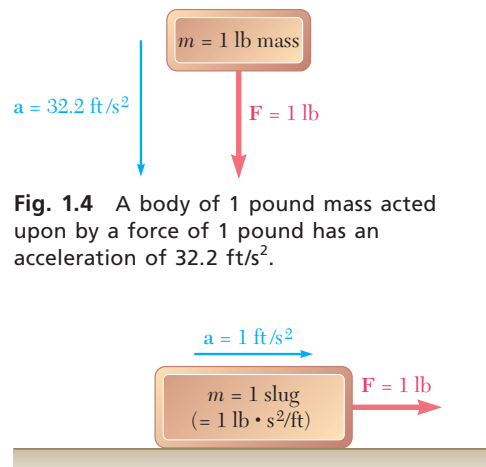
$$1 \text{ slug} = \frac{1 \text{ lb}}{1 \text{ ft/s}^2} = 1 \text{ lb} \cdot \text{s}^2/\text{ft} \quad (1.6)$$

Comparing Figs. 1.4 and 1.5, we conclude that the slug is a mass 32.2 times larger than the mass of the standard pound.

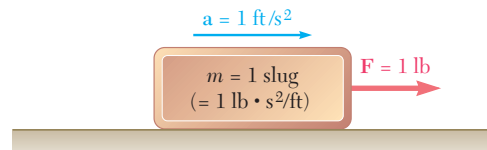
The fact that, in the U.S. customary system of units, bodies are characterized by their weight in pounds rather than by their mass in slugs is convenient in the study of statics, where we constantly deal with weights and other forces and only seldom deal directly with masses. However, in the study of dynamics, where forces, masses, and accelerations are involved, the mass  $m$  of a body is expressed in slugs when its weight  $W$  is given in pounds. Recalling Eq. (1.4), we write

$$m = \frac{W}{g} \quad (1.7)$$

where  $g$  is the acceleration due to gravity ( $g = 32.2 \text{ ft/s}^2$ ).



**Fig. 1.4** A body of 1 pound mass acted upon by a force of 1 pound has an acceleration of  $32.2 \text{ ft/s}^2$ .



**Fig. 1.5** A force of 1 pound applied to a body of mass 1 slug produces an acceleration of  $1 \text{ ft/s}^2$ .